NAG Toolbox for MATLAB

g13aj

1 Purpose

g13aj applies a fully specified seasonal ARIMA model to an observed time series, generates the state set for forecasting and (optionally) derives a specified number of forecasts together with their standard deviations.

2 Syntax

```
[rms, st, nst, fva, fsd, isf, ifail] = g13aj(mr, par, c, kfc, x, ist,
nfv, ifv, 'npar', npar, 'nx', nx)
```

3 Description

The time series $x_1, x_2, \ldots, x_n$ supplied to the function is assumed to follow a seasonal autoregressive integrated moving average (ARIMA) model with known parameters.

The model is defined by the following relations.

(a) $\nabla^d \nabla_s^D x_t - c = w_t$ where $\nabla^d \nabla_s^D x_t$ is the result of applying non-seasonal differencing of order d and seasonal differencing of seasonality s and order D to the series x_t, and c is a constant.

(b) $w_t = \Phi_1 w_{t-s} + \Phi_2 w_{t-2\times s} + \cdots + \Phi_P w_{t-P\times s} + e_t - \Theta_1 e_{t-s} - \Theta_2 e_{t-2\times s} - \cdots - \Theta_Q e_{t-Q\times s}$.

This equation describes the seasonal structure with seasonal period s; in the absence of seasonality it reduces to $w_t = e_t$.

(c) $e_t = \phi_1 e_{t-1} + \phi_2 e_{t-2} + \cdots + \phi_p e_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q}$.

This equation describes the non-seasonal structure.

Given the series, the constant c, and the model parameters Φ, Θ, ϕ, θ, the function computes the following.

(a) The state set required for forecasting. This contains the minimum amount of information required for forecasting and comprises:

(i) the differenced series w_t, for $(N - s \times P) \leq t \leq N$;

(ii) the $(d + D \times s)$ values required to reconstitute the original series x_t from the differenced series w_t;

(iii) the intermediate series e_t, for $N - \max(p, Q \times s) < t \leq N$;

(iv) the residual series a_t, for $(N - q) < t \leq N$, where $N = n - (d + D \times s)$.

(b) A set of L forecasts of x_t, for $t = n + 1, n + 2, \ldots, n + L$ (L may be zero).

The forecasts are generated from the state set, and are identical to those that would be produced from the same state set by g13ah.

Use of g13aj should be confined to situations in which the state set for forecasting is unknown. Forecasting from the series requires recalculation of the state set and this is relatively expensive.

4 References

Box G E P and Jenkins G M 1976 *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden–Day

5 Parameters

5.1 Compulsory Input Parameters

1: **mr**(7) – **int32 array**

The orders vector (p, d, q, P, D, Q, s) of the ARIMA model, in the usual notation.

Constraints:

$p, d, q, P, D, Q, s \geq 0$;
$p + q + P + Q > 0$;
$s \neq 1$;
if $s = 0$, $P + D + Q = 0$;
if $s > 1$, $P + D + Q > 0$.

2: **par**(**npar**) – **double array**

The p values of the ϕ parameters, the q values of the θ parameters, the P values of the Φ parameters, and the Q values of the Θ parameters, in that order.

3: **c** – **double scalar**

c, the expected value of the differenced series (i.e., c is the constant correction). Where there is no constant term, **c** must be set to 0.0.

4: **kfc** – **int32 scalar**

Must be set to 0 if **c** was not estimated, and 1 if **c** was estimated. This is irrespective of whether or not **c** = 0.0. The only effect is that the residual degrees of freedom are one greater when **kfc** = 0. Assuming the supplied time series to be the same as that to which the model was originally fitted, this ensures an unbiased estimate of the residual mean-square.

Constraint: **kfc** = 0 or 1.

5: **x**(**nx**) – **double array**

The n values of the original, undifferenced time series.

6: **ist** – **int32 scalar**

Constraint: **ist** $\geq (P \times s) + d + (D \times s) + q + \max(p, Q \times s)$. The expression on the right-hand side of the inequality is returned in **nst**.

7: **nfv** – **int32 scalar**

The required number of forecasts. If **nfv** ≤ 0, no forecasts will be computed.

8: **ifv** – **int32 scalar**

Constraint: **ifv** $\geq \max(1, \mathbf{nfv})$.

5.2 Optional Input Parameters

1: **npar** – **int32 scalar**

Default: The dimension of the array **par**.

The exact number of ϕ, θ, Φ and Θ parameters.

Constraint: **npar** $= p + q + P + Q$.

2: **nx** – **int32 scalar**

Default: The dimension of the array **x**.

n, the length of the original, undifferenced time series.

5.3 Input Parameters Omitted from the MATLAB Interface

w, iw

5.4 Output Parameters

1: **rms – double scalar**

The residual variance (mean square) associated with the model.

2: **st**(**ist**) **– double array**

The **nst** values of the state set.

3: **nst – int32 scalar**

The number of values in the state set array **st**.

4: **fva**(**ifv**) **– double array**

If **nfv** > 0, **fva** contains the **nfv** forecast values relating to the original undifferenced time series.

5: **fsd**(**ifv**) **– double array**

If **nfv** > 0, **fsd** contains the estimated standard errors of the **nfv** forecast values.

6: **isf**(**4**) **– int32 array**

Contains validity indicators, one for each of the four possible parameter types in the model (autoregressive, moving average, seasonal autoregressive, seasonal moving average), in that order.

Each indicator has the interpretation:

-1 On entry the set of parameter values of this type does not satisfy the stationarity or invertibility test conditions.

0 No parameter of this type is in the model.

1 Valid parameter values of this type have been supplied.

7: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **npar** $\neq p + q + P + Q$,
or the orders vector **mr** is invalid (check the constraints in Section 5),
or **kfc** $\neq 0$ or 1.

ifail $= 2$

On entry, **nx** $- d - D \times s \leq$ **npar** $+$ **kfc**, i.e., the number of terms in the differenced series is not greater than the number of parameters in the model. The model is over-parameterised.

ifail $= 3$

On entry, the workspace array **w** is too small.

ifail $= 4$

On entry, the state set array **st** is too small. It must be at least as large as the exit value of **nst**.

ifail $= 5$

This indicates a failure in f04as which is used to solve the equations giving estimates of the backforecasts.

ifail $= 6$

On entry, valid values were not supplied for all parameter types in the model. Inspect array **isf** for further information on the parameter type(s) in error.

ifail $= 7$

On entry, **ifv** $< \max(1, \mathbf{nfv})$.

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by g13aj is approximately proportional to n and the square of the number of backforecasts derived.

9 Example

```
mr = [int32(1);
     int32(1);
     int32(2);
     int32(0);
     int32(0);
     int32(0);
     int32(0)];
par = [-0.0547;
     -0.5568;
     -0.6636];
c = 9.980700000000001;
kfc = int32(1);
x = [-217;
     -177;
     -166;
     -136;
     -110;
     -95;
     -64;
     -37;
     -14;
     -25;
     -51;
     -62;
     -73;
     -88;
     -113;
     -120;
     -83;
     -33;
     -19;
     21;
     17;
     44;
     44;
```

```
     78;
     88;
     122;
     126;
     114;
     85;
     64];
ist = int32(4);
nfv = int32(5);
ifv = int32(5);
[rms, st, nst, fva, fsd, isf, ifail] = g13aj(mr, par, c, kfc, x, ist,
nfv, ifv)
```

```
rms =
  375.9146
st =
   64.0000
  -30.9807
  -20.4495
   -2.7212
nst =
           4
fva =
   60.5899
   69.4973
   79.5367
   89.5142
   99.4951
fsd =
   19.3885
   34.9870
   54.2475
   67.8676
   79.1975
isf =
           1
           1
           0
           0
ifail =
           0
```